Relaxation Phenomena in the Adiabatic Phase Transition of Type I Superconductor Particles

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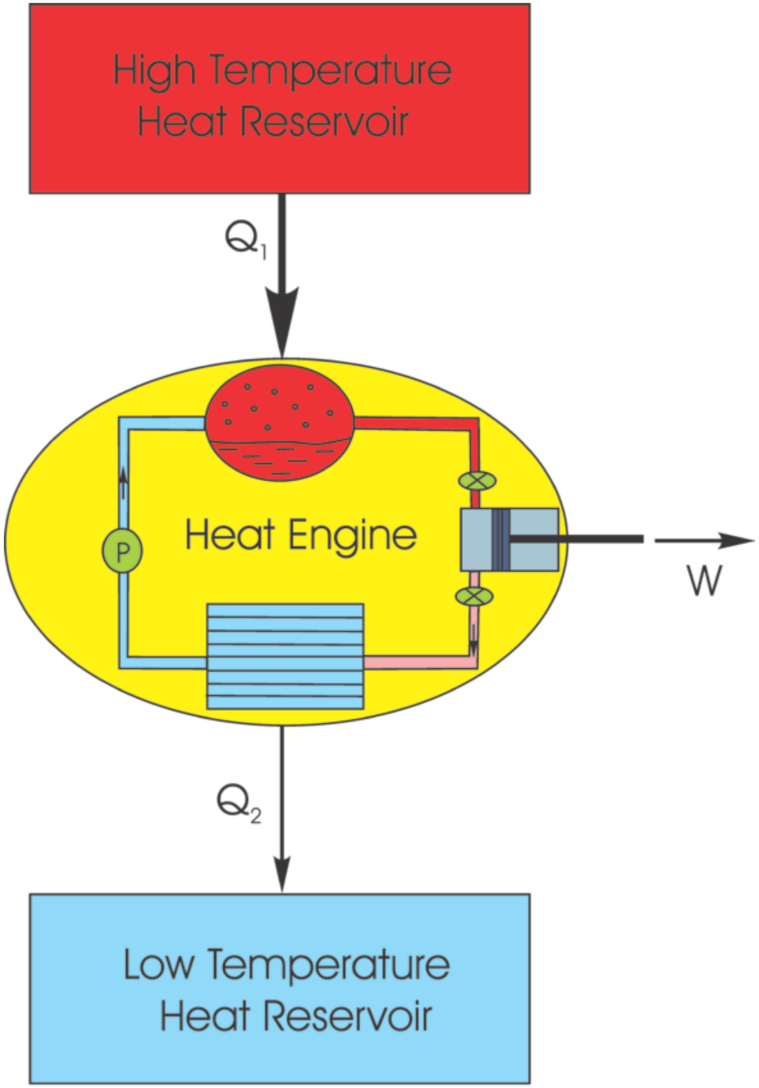
Classical Heat Engine



$$Q_1 = Q_2 + W$$

Heat, Q_1, boils water in the boiler. Resulting steam actuates a piston of a valved cylinder, producing work, W. The cooled steam then passes through a condenser where heat, Q_2, is rejected to the atmosphere. Resulting water is then pumped back to the boiler.

Properties of Type I Superconductors

- **Perfect electrical conductor**
- **Perfect diamagnet**
- **Entropy of the superconductive phase is less than the entropy of the normal phase**
- **First order phase transition**

  A latent heat is absorbed at the phase transition from superconductive to normal phase because of the difference in entropy of the phases.

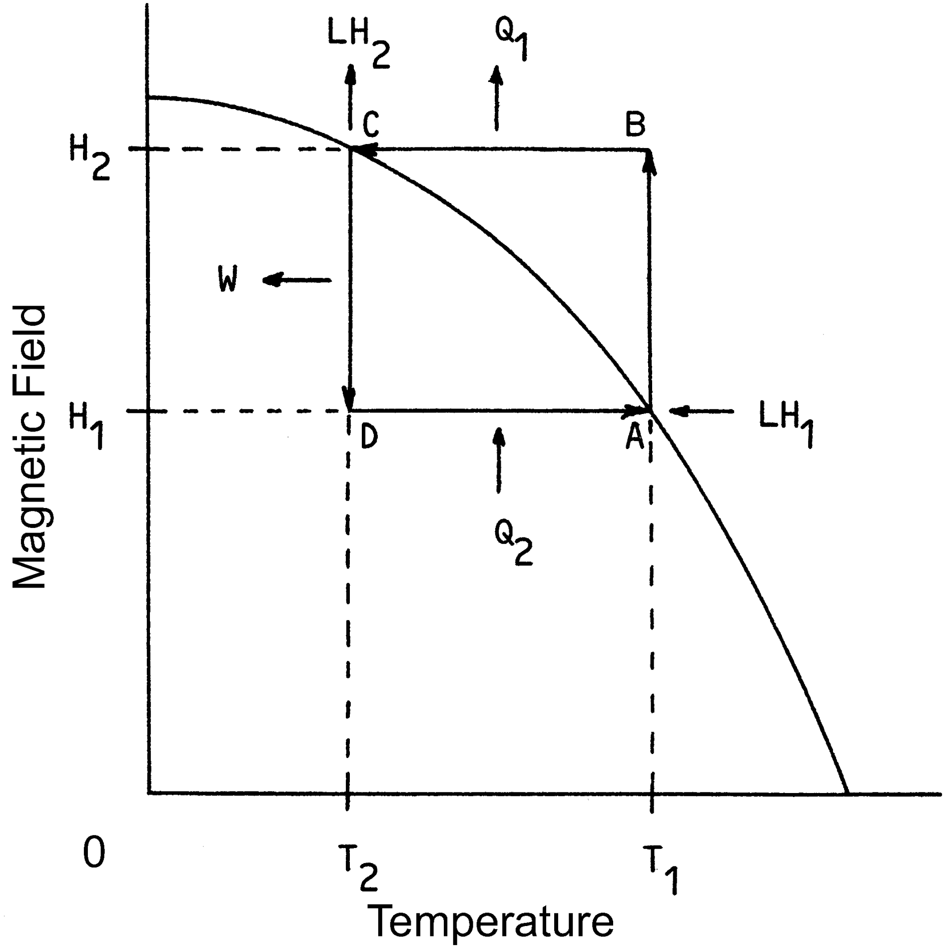
- **Magneto-Caloric Effect**

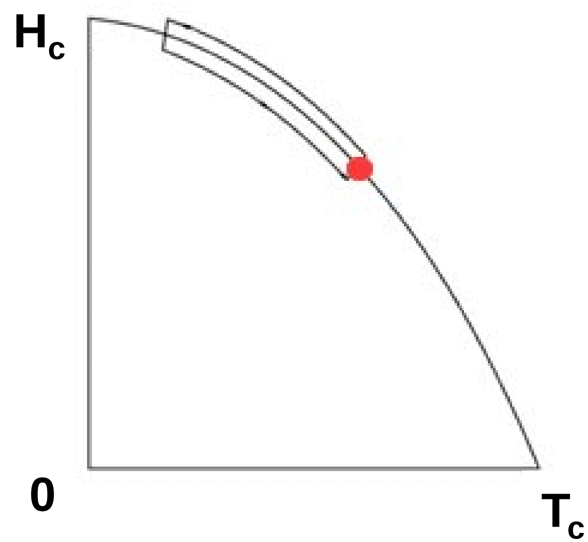
  An adiabatically isolated superconductor driven normal by a magnetic field will self-cool because of the first order phase transition.

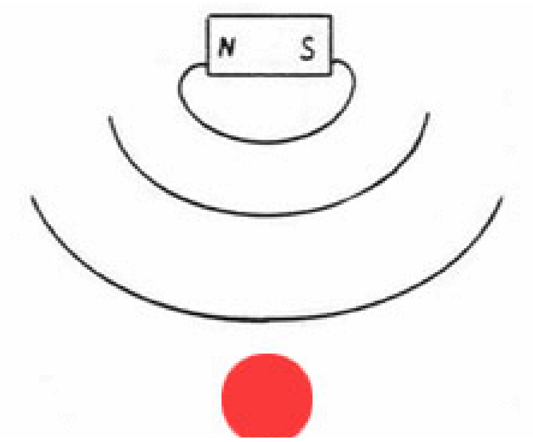
- **Meissner Effect**

  Expulsion of magnetic field occurs by merely lowering the temperature.

Heat Engine Using a Bulk Superconductor as the Working Media







$$W_{out} = Q_{in} = (LH_1 - LH_2) + (Q_2 - Q_1)$$

$$\eta = [1 - (Q_1 + LH_2)/(Q_2 + LH_1)]x100$$

normal phase

superconductive phase

The Magneto-Caloric Effect

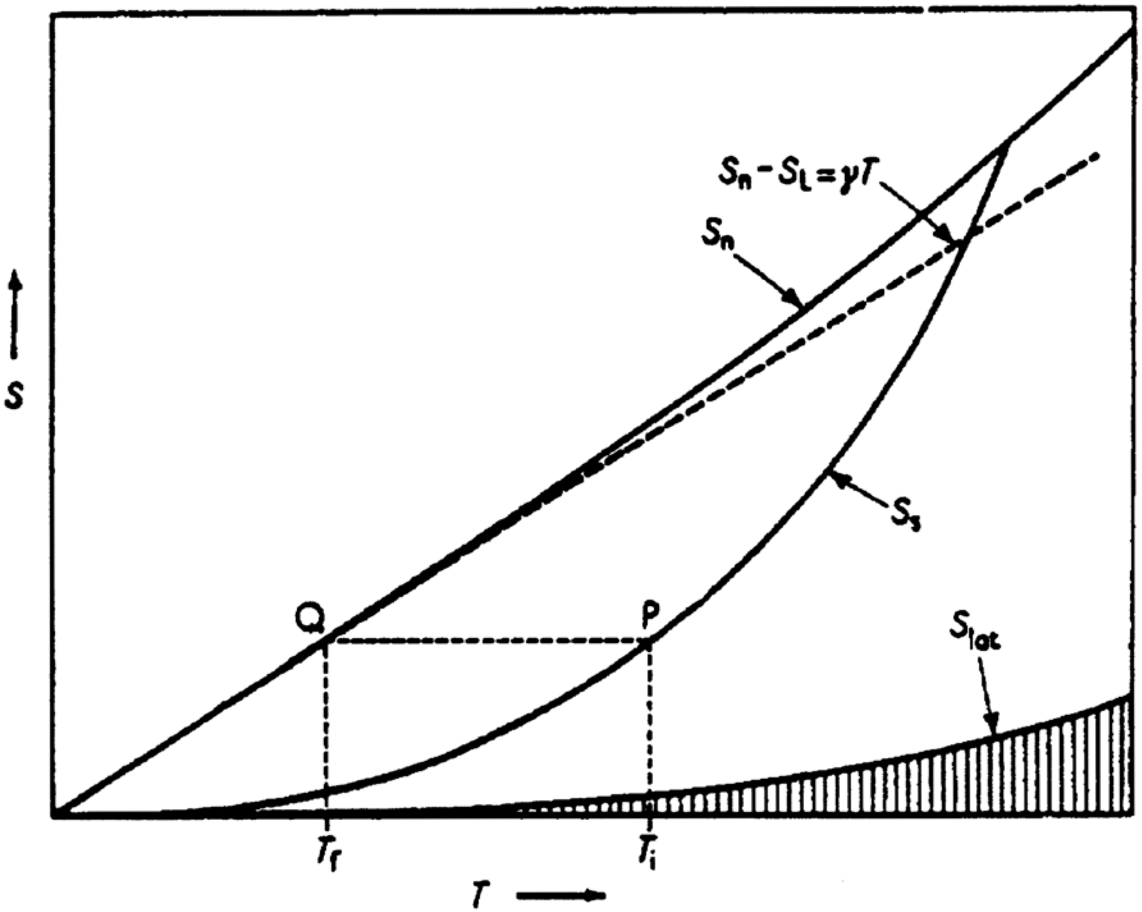


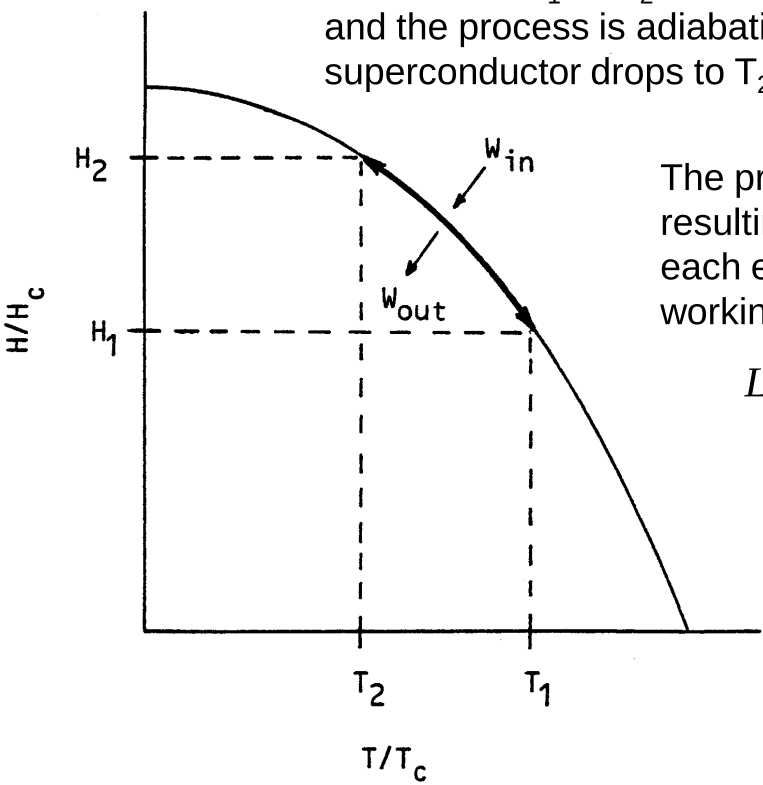
Figure 1. Diagram of entropy S versus temperature T curve

Taken from M. Yaqub, Cryogenics, December, 1960, 101

When a Type I Superconductor in the superconductive phase is driven normal by an increasing magnetic field, the phase transition is first order, and the temperature drops. The process is isentropic, and the final temperature is determined by the isentrop P-Q.

Adiabatic Phase Transition Process Performed on a Bulk Size (d >> ξ) Superconductor Working Media

Beginning at T_1 in the superconductive phase, the magnetic field is raised from H_1 to H_2. Because the phase transition is first order and the process is adiabatic, the temperature of the superconductor drops to T_2.

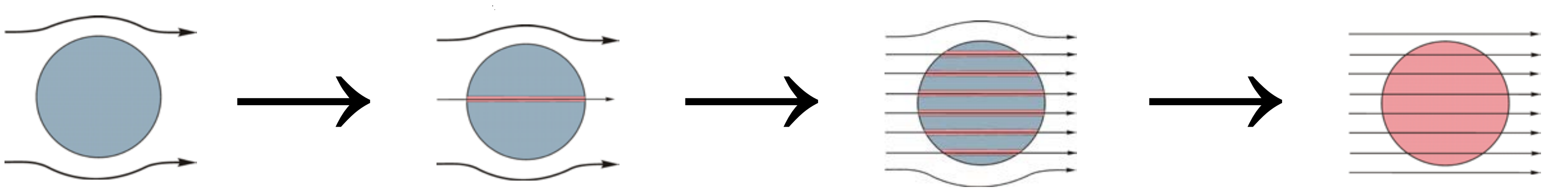


The process is performed infinitely slowly in increments of ΔH, resulting in incremental phase volume changes, ΔV, and for each elemental volume change, the latent heat cools the working medium by the relation:

$$LH_{\Delta V} = \int_{T_a}^{T_b} (C_n dT)(V_n + \Delta V) + \int_{T_a}^{T_b} (C_s dT)(V_s - \Delta V)$$

The process is reversed from T_2 to T_1.

Magnetic work, W_{in}, is input and magnetic work, W_{out}, is output, where $W_{in} = W_{out}$ due to the in-process presence of superconductive phase in the intermediate state.



Superconductive phase Intermediate State Normal phase

The Magneto-Caloric Effect

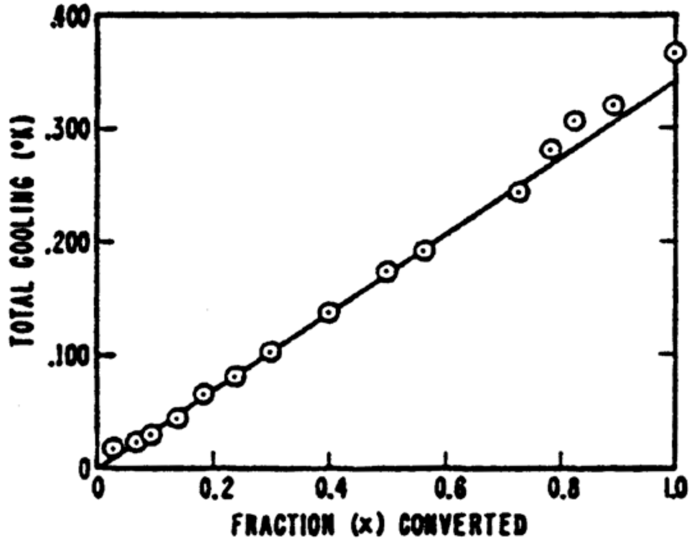
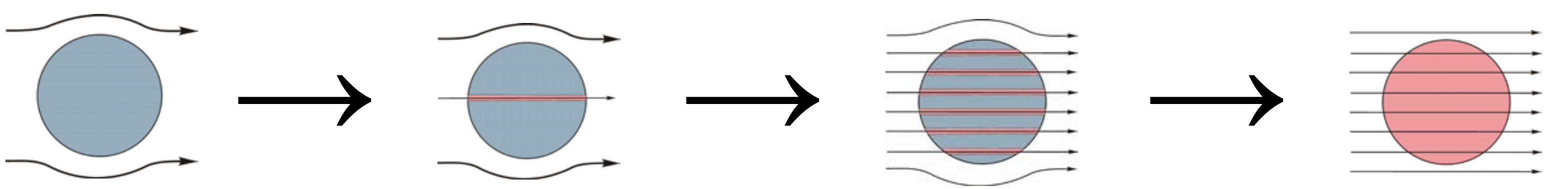


Fig. 2. Demonstration of the linear relationship between the applied magnetic field and the fraction of normal metal produced in the intermediate state. The straight line is drawn with a slope to fit best the data $x \leq 0.6$ for which eddy-current corrections are small.

Taken from R. Dolecek, Phy. Rev. 96, 25 (1954)



Superconductive phase

Intermediate State

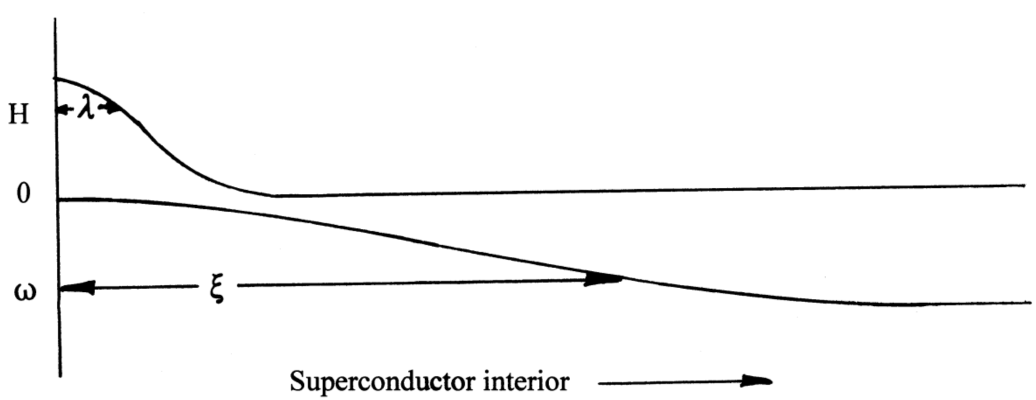
Normal phase

Size Scale Regimes

- **Macroscopic Regime**
  - The cross-section of the working media is much larger than the coherence length, >.
  - Ensemble averaging results in relaxation time differences of the phase space variables being unobservable.

- **Mesoscopic Regime**
  - The cross-section of the working media is about equal to the coherence length, >.
  - Homogeneous coherence results in relaxation time differences of the phase space variables being observable.

- **Microscopic Regime**
  - The cross-section of the working media is much smaller than the coherence length >.
  - Vanishing dimensionality results in relaxation time differences of the phase space variables being unobservable.



8 represents the "*penetration depth*" of the applied magnetic field, H, at the surface of the superconductor. 8 is on the order of about 10^{-5} cm.

> represents the "*coherence length*" of the order parameter, T, of the superelectrons. > is on the order of about 10^{-4} cm.

Size and the Intermediate State

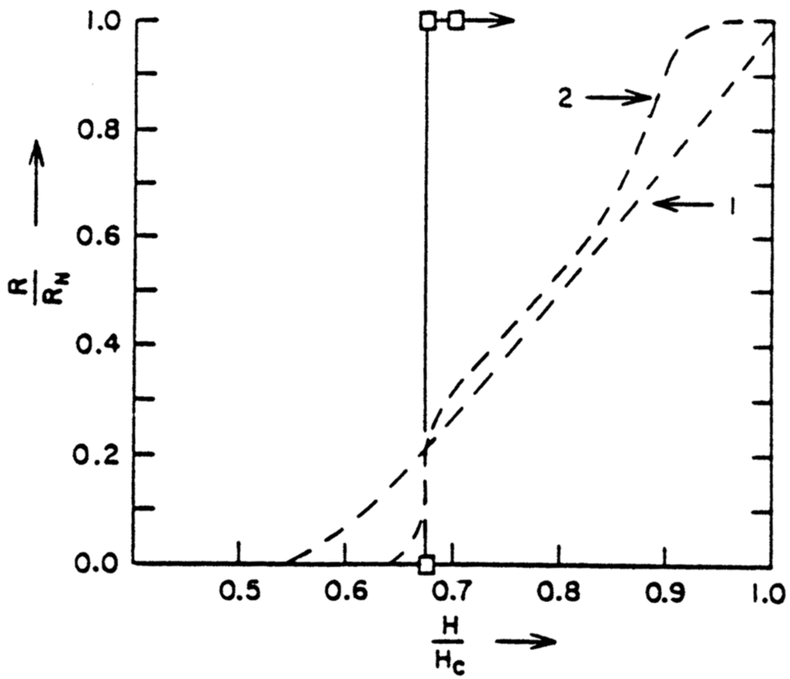
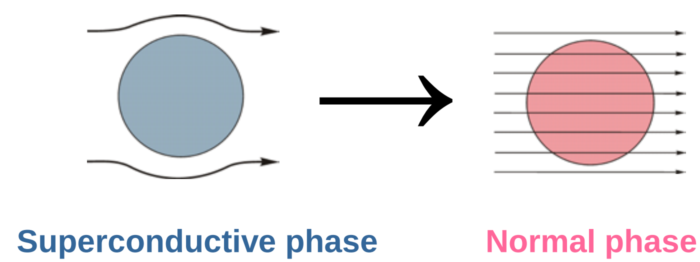


FIG. 2. Comparison of whisker transition with results of Andrew for larger wires. 1—Andrew, 1.05×10^{-1} cm diameter, 1.66°K. 2—Andrew, 27×10^{-4} cm diameter, 1.66°K. □—whisker, 1.2×10^{-4} cm diameter, 1.69°K.

Taken from O. Lutes, E. Maxwell, Phy. Rev. 97, 1718 (1955)

Lutes and Maxwell discovered that for thin cylinders (whiskers) of tin having N=1/2 and diameter near >(T), an intermediate state is not observed.



Size and Specific Heat

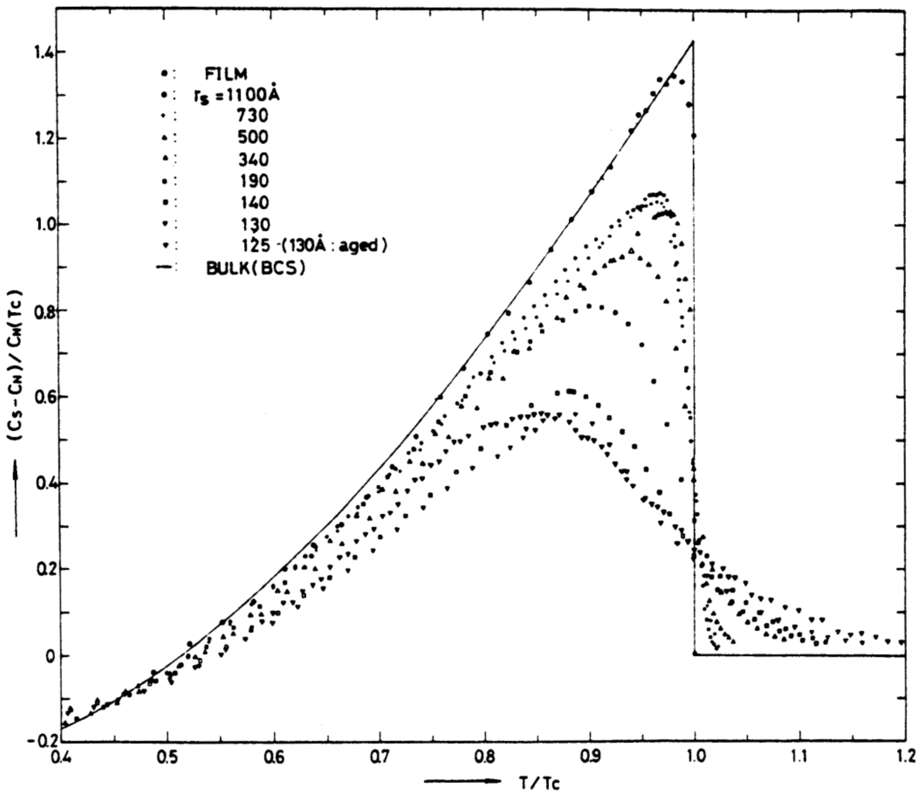


Fig. 5. Specific heat difference for Sn particles with different particle sizes as a function of the reduced temperature. The difference is normalized to $C_N(T_c)$, where $C_N(T_c)=\gamma T_c$ with $\gamma=1.78\ \mathrm{mJ\cdot K^{-2}\cdot mol^{-1}}$.

Tsuboi and Suzuki report that for cross-sections in the range of about d = >(T), the specific heat function remains that applicable to bulk specimens.

Taken from T. Tsuboi, T. Suzuki, J. Phy. Soc. Jap., 42, 437 (1977)

Size and First Order Phase Transition

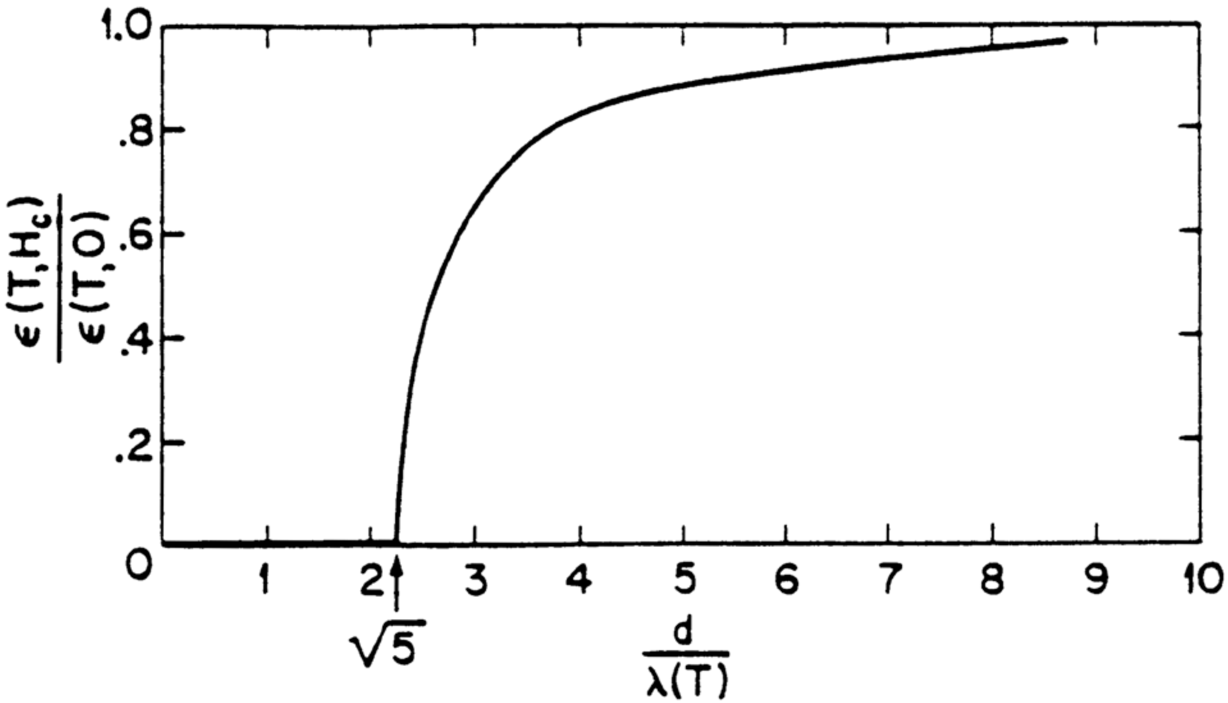


FIG. 1. Energy gap at the critical field vs thickness.

Taken from D. H. Douglass, Phy. Rev. Lett. 6, 346 (1961)

Douglass predicts that upon application of H_c, the phase transition can be second order with no latent heat for cross-sections, d, of d <√58(T), or be first order with a reduced latent heat for cross-sections of √58(T) < d < 58(T).

Relaxation Times of the State Variables (T, H)

- **Thermal Relaxation (T)**

  At very low temperature, heat transfer through the working media is mainly mediated by phonons through the lattice and minimally by the conduction electrons. In Al, an upper limit of phonon propagation[1] is about $6.5\text{x}10^{3}$ m/s, therefore requiring about 10^{-8} seconds to travel a distance **>**.

- **Electrodynamic Relaxation (H)**

  The rate of magnetic flux movement is related to the speed of electromagnetic (E-M) waves through the working media. In Al, a lower limit of E-M wave propagation[2] is about $1.5\text{x}10^{6}$ m/s, therefore requiring about 10^{-11} seconds to travel a distance **>**.

For the macroscopic and microscopic regimes, where the phase transition is characterized by equilibrium processes, the thermal and electrodynamic relaxations are considered to be simultaneous and coupled, occurring[3] between about 10^{-8} and 10^{-11} seconds.

For the mesoscopic regime, where the phase transition is characterized by non-equilibrium processes, the thermal and electrodynamic relaxations are considered to be non-simultaneous and decoupled, where the electrodynamic relaxation time is several orders of magnitude faster than the thermal relaxation time.

1. CRC Handbook of Chem. and Phys., 51st Ed., pg. E-41.
2. Measurement Science Rev., V.4., Sec. 3 (2004).
3. Prog. in Low Temp. Phys., VI, 172 (1955).

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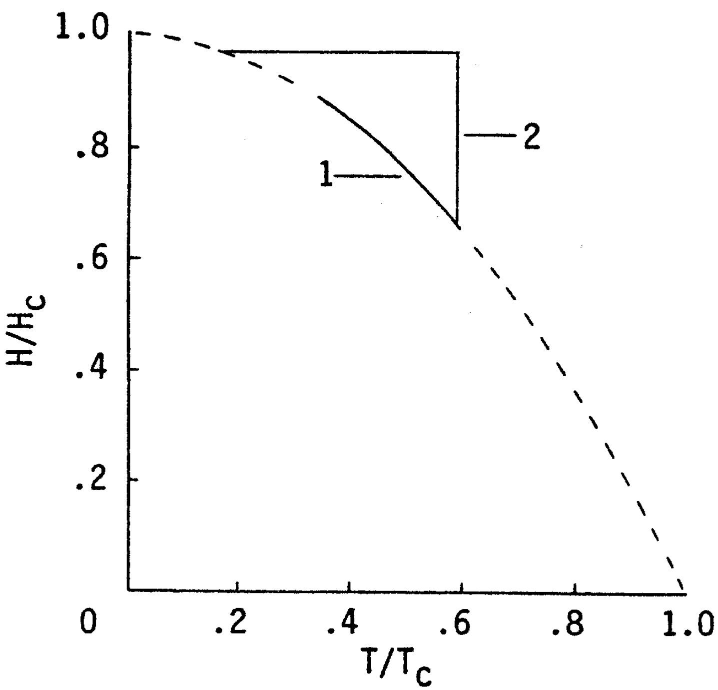
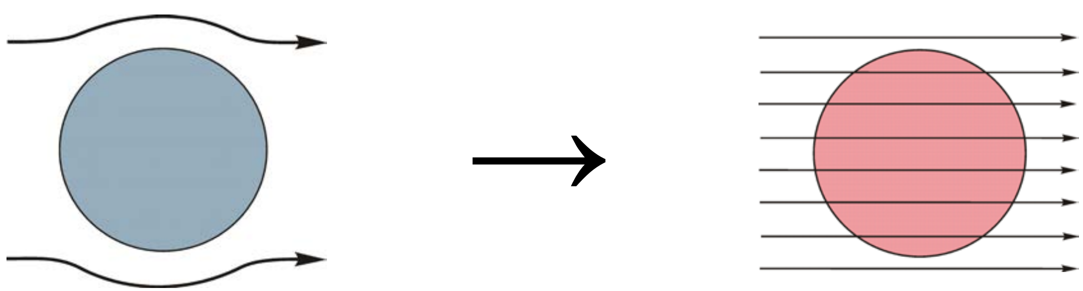
Adiabatic Phase Transition Process Performed on a Particle Size (d = ≳) Superconductor Working Media



<u>Curve 1</u> represents the macroscopic adiabatic demagnetization process, performed at an infinitely slow rate so that the ensemble entropy remains constant.

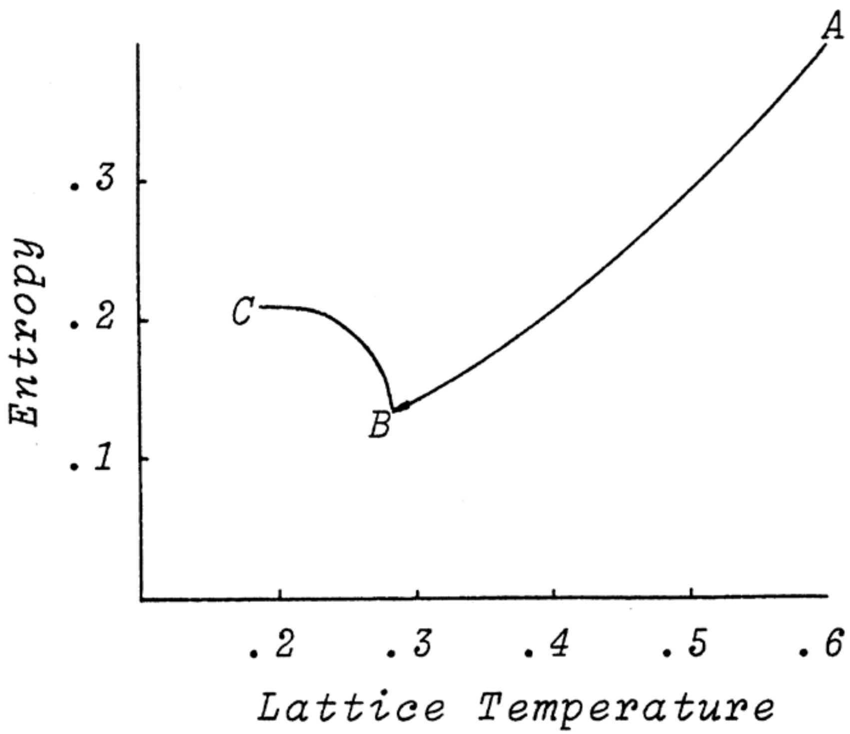
<u>Curve 2</u> represents the mesoscopic adiabatic demagnetization process, performed at a maximal rate allowed by the difference in thermal and electrodynamic relaxation times.



Superconductive phase

Normal phase

Experiment Theory



Point A: Starting at T_1 = .6 T_c, in the superconductive phase.

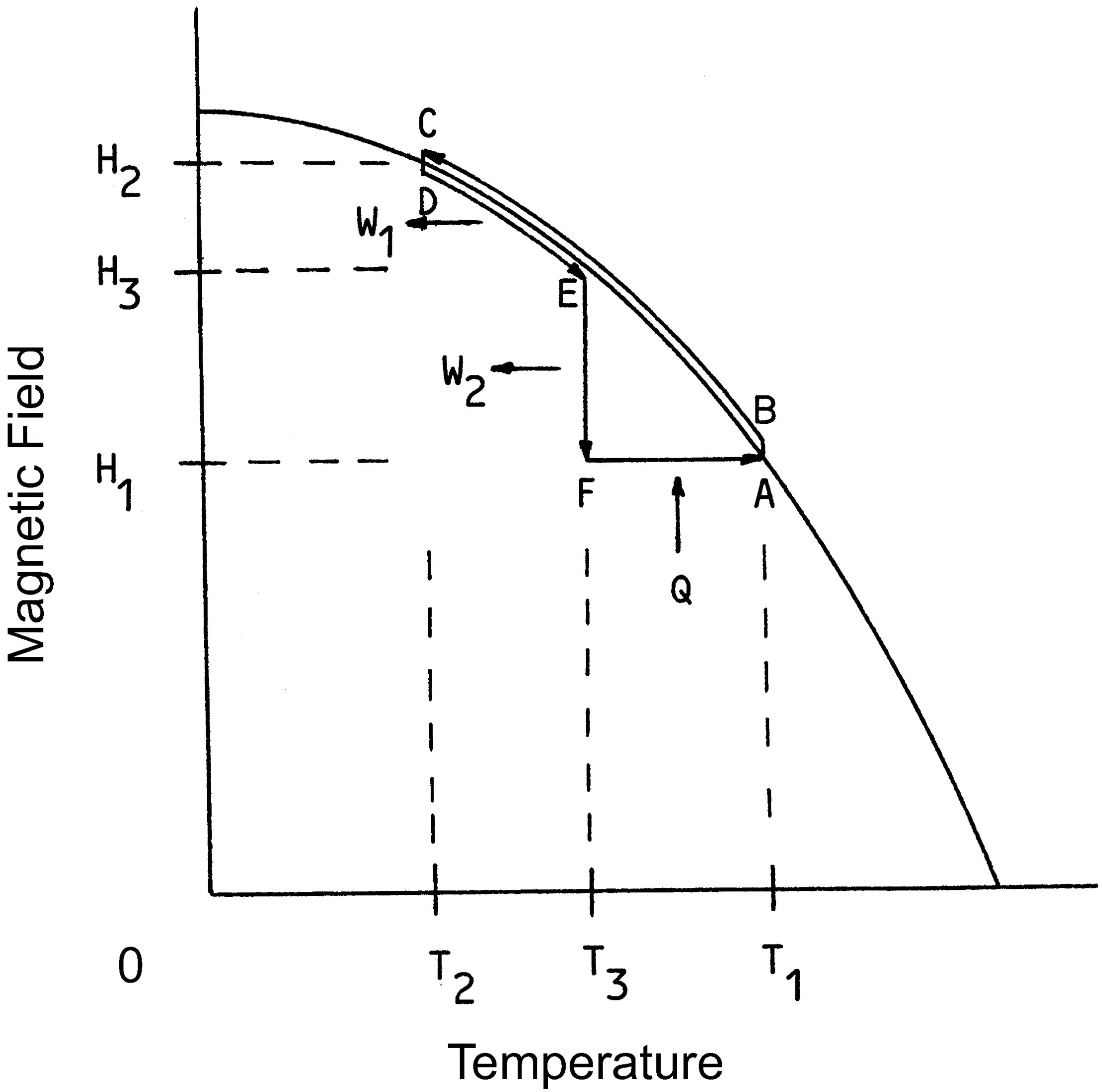
Point A to Point B: Superelectrons below the Fermi sea gain entropy from the lattice and normal conduction electrons; overall entropy decreases.

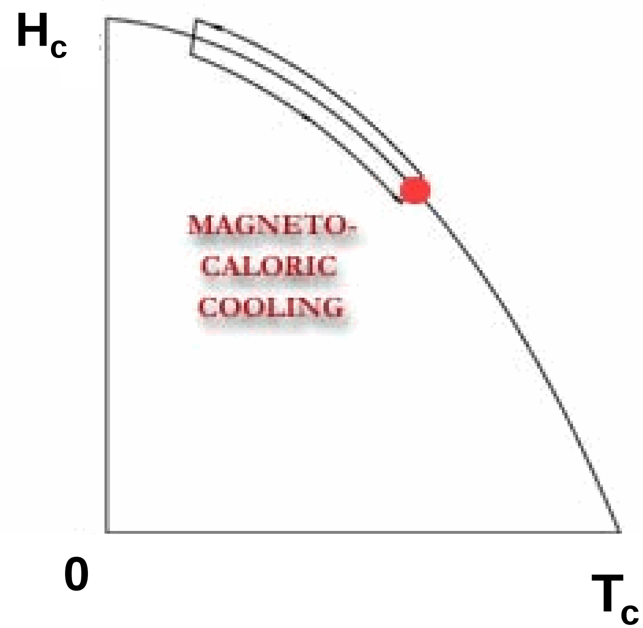
Point B to Point C: Superelectrons now above the Fermi sea continue to gain entropy from the lattice and normal conduction electons; overall entropy increases.

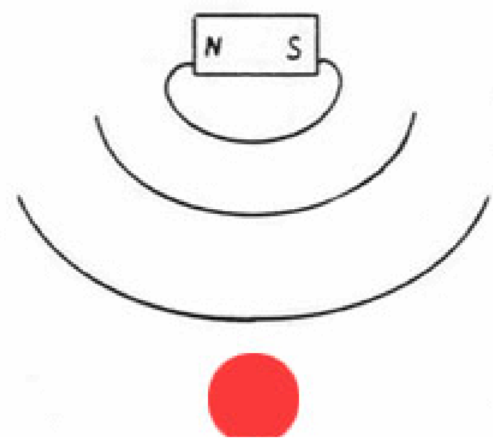
Point C: Ending at T'_2 = .186T_c, in the normal phase. The ending entropy is lower than the starting entropy, in violation of conventional formulations of the 2nd Law.

Temperature is in terms of T/T_c and entropy is in arbitrary units.

Coherent Magneto-Caloric Effect Magnetization Process Cycle

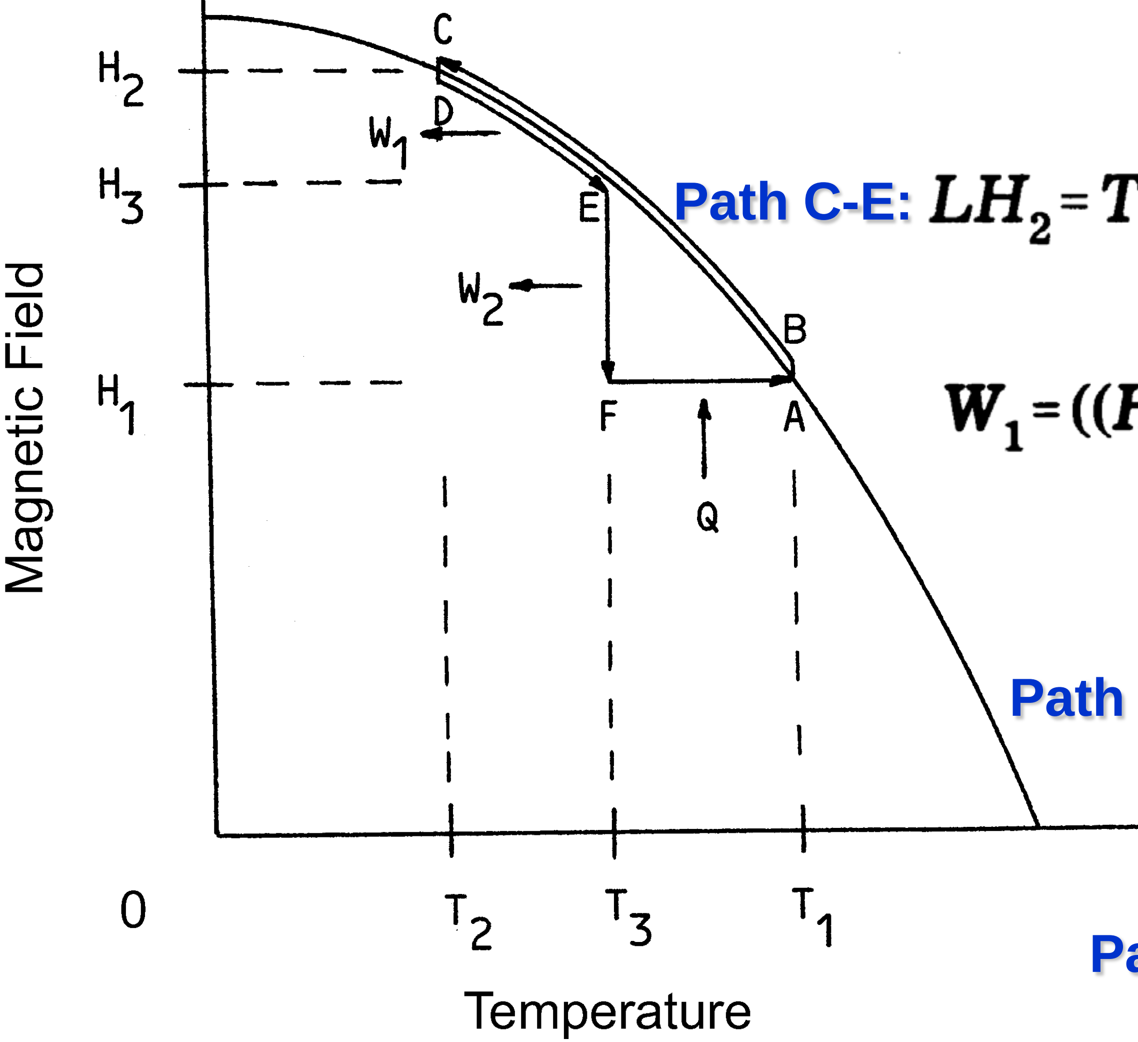






normal phase

superconductive phase



Path A-C: $LH_1 = T_1(S_{n1} - S_{s1}) = \left(\int_{T_1}^{T_2} C_n\, dT\right)\cdot V$

Path C-E: $LH_2 = T_2(S_{n2} - S_{s2}) = \left(\int_{T_2}^{T_3} C_s\, dT\right)\cdot V$

$W_1 = ((H_2^2 - H_3^2)/8\pi)\cdot V$

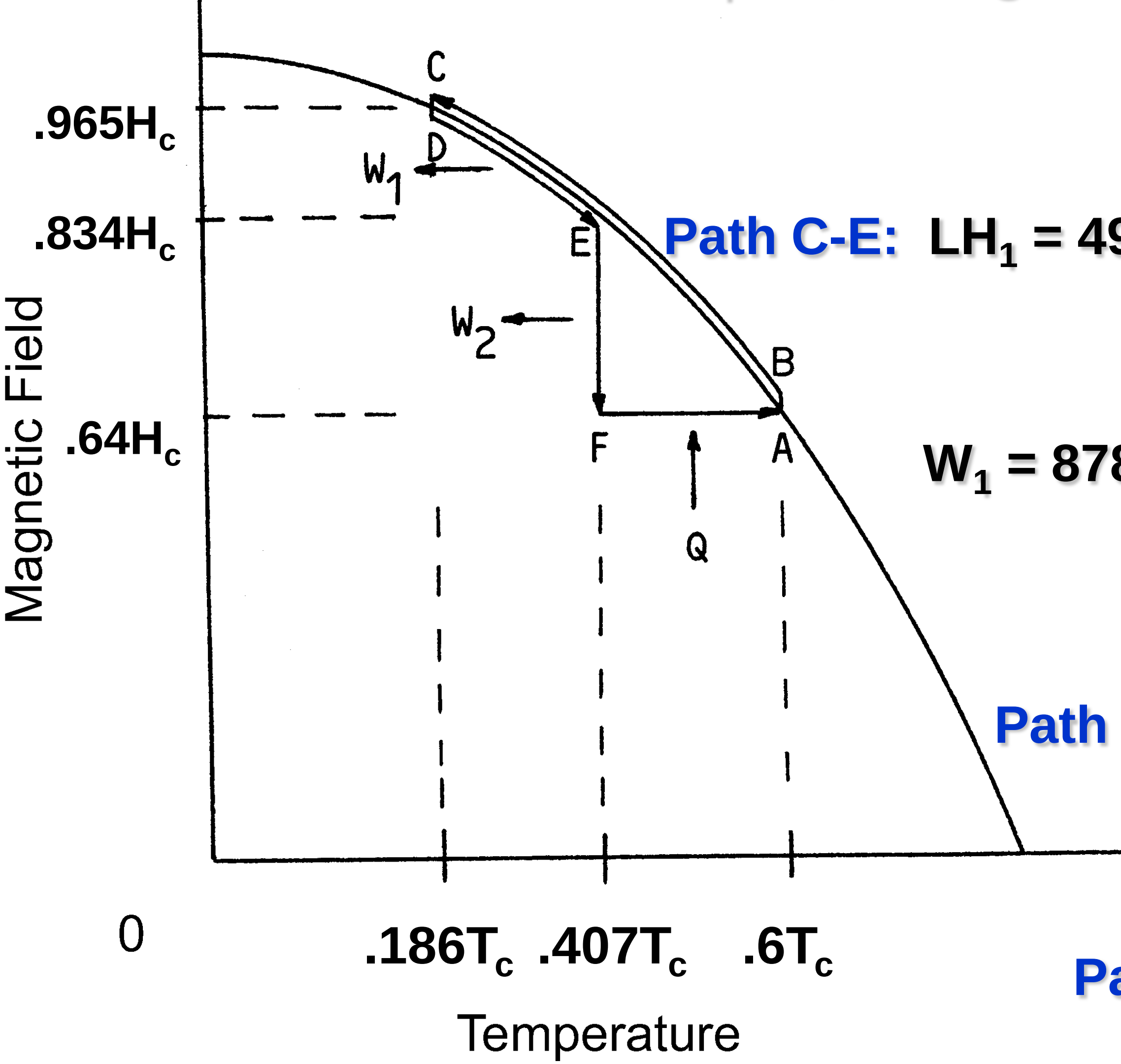
Path E-F: $W_2 = ((H_3^2 - H_1^2)/8\pi)\cdot V$

Path F-A: $Q = \left(\int_{T_3}^{T_1} C_s\, dT\right)\cdot V$

$$Q = W_1 + W_2$$

Example: Sn armature

Path A-C: LH_1 = 3,433 ergs/cm^3



Path C-E: LH_1 = 499 ergs/cm^3

W_1 = 878.5 ergs/cm^3

Path E-F: W_2 = 1,066.5 ergs/cm^3

Path F-A: Q = 1,945 ergs/cm^3

$$Q = W_1 + W_2 = 1{,}945 \text{ ergs/cm}^3$$

An intermediate state is not possible if the cross-section of the armature, d, is such that **>(T) $d**.

A first order phase transition requires **d $√58(T)**, preferably **d $58(T)**.

Possible **>(T) $d $58(T)** superconductors:

aluminum, where >/8 = 32

indium, where >/8 = 6.875

tin, where >/8 = 4.5

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Urbana
Illinois 61801

March 25, 1987

Dr. Peter D. Keefe
E. E. & Physics Faculty
4001 West McNichols Road
Detroit, MI 48221-9987

Dear Dr. Keefe:

At long last I have found a little time to study your proposal. The reprints of the earlier literature have been very helpful. However, I am puzzled by the cooling step that you say involves no work and no heat input. It should be possible to go from the superconducting phase at T_1, H_1 to a normal phase at T_2, but then one would have to reduce the field at T_2 to bring it into the superconducting phase. The adiabatic transition from T_1, H_1 to T_2, H_2 would require an input of work in the cooling step and there would be dissipation of heat in going from the normal to superconducting phases at T_2. The work input at the cooling step would be greater than the work output at the heating step, resulting in a net input of work going into heat.

Apparently you would like to have an adiabatic step from the superconducting phase at T_1, H_1 to normal at T_2, H_2, but I don't see how this can happen without violating the laws of thermodynamics. Further the minimum applied field, H_a, must satisfy

$$F_s(T_1) + \frac{H_a^2}{8\pi} = F_n(T_2)$$

or

$$\frac{H_a^2}{8\pi} = F_n(T_2) - F_s(T_1)$$

with the condition $S_s(T_1) = S_n(T_2)$. Since $F_s(T_1) < F_s(T_2)$, H_a must be greater than H_2. This implies that the transition cannot take place at H_1, but there must be considerable "superheating" to the higher field H_a. When at T_2 the field must be reduced below H_2 to make the transition to the superconducting phase. Heat is then released to the low temperature resevoir. Since the transition takes place at H_a, work that must be done on the system as H_1 increases to H_a. This is more than the external work obtained as the system is heated.

Perhaps I have misinterpreted your ideas, but it seems to me that it is the assumption of no superheating that is at fault. It is a long time since I have thought about the thermodynamics of superconductors.

Sincerely,

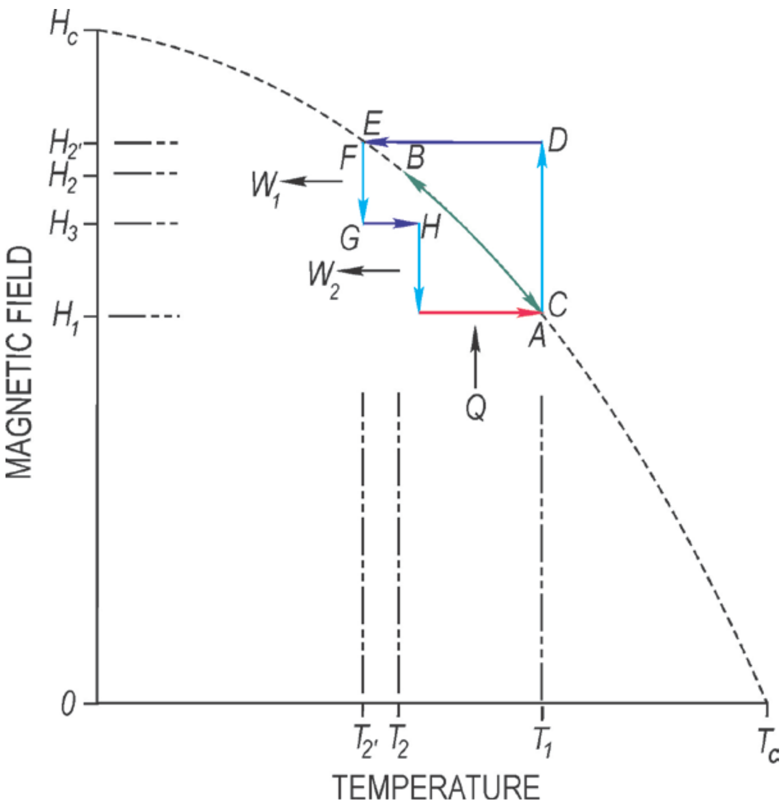
John Bardeen

John Bardeen

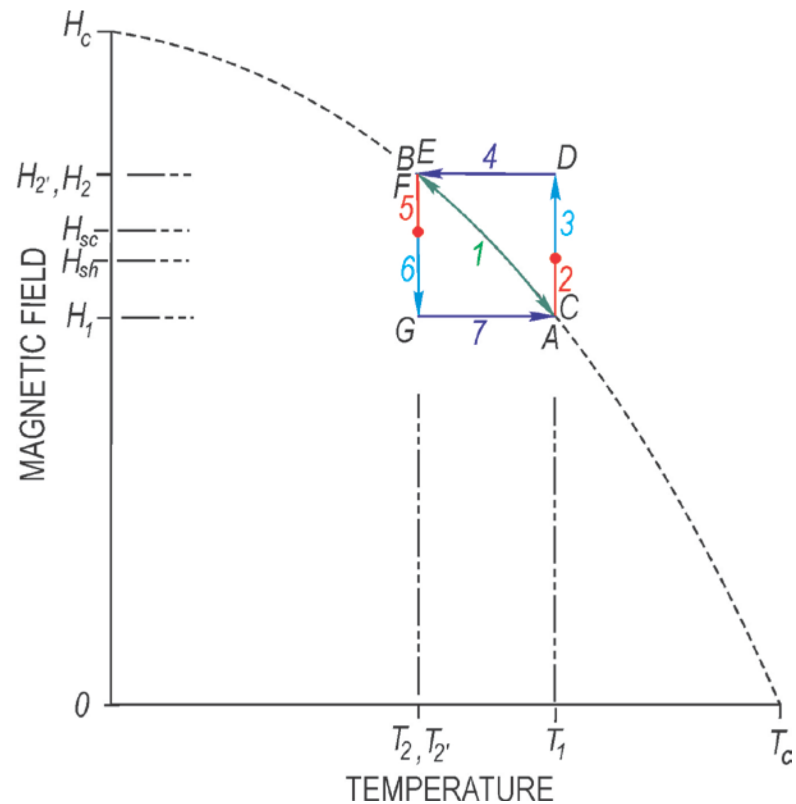
John Bardeen

Private communication
to the Author, 03/25/1987

Bardeen’s Hypothesis of Superheating and Supercooling Fields



No Magnetic Hysteresis



Bardeen’s Magnetic Hysteresis

Magnetic Hysteresis in a Mesoscopic Type I Superconductor

XXI. *Magnetic Hysteresis in Superconducting Colloids*

By A. B. Pippard
Royal Society Mond Laboratory, Cambridge*

[Received November 2, 1951]

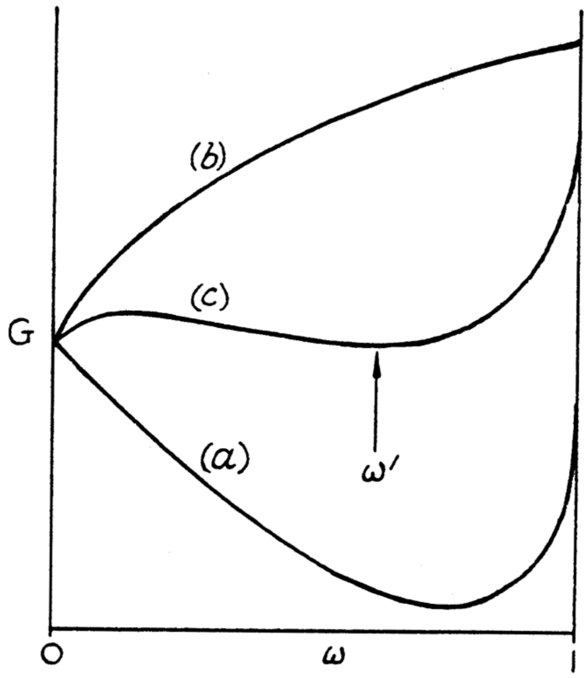
Abstract

The irreversible effects exhibited in the magnetization cycle of a superconducting colloid are analysed in terms of a simple model of a superconductor. Expressions are derived relating various critical field strengths to temperature and radius of the particle; there is fair agreement between experiment and theory.

Taken from A. B. Pippard, Phil. Mag., 43, 273 (1952)

Origin of Superheating in a Mesoscopic Type I Superconductor

Fig. 1



Gibbs function of small particle :
(a) in absence of field,
(b) field contribution,
(c) resultant curve for G.

Taken from A. B. Pippard, Phil. Mag., 43, 273 (1952)

Pippard suggests that the Gibbs function (curve c) is related to two contributions:

1) The ordering of the electrons, ω, (curve a) in absence of magnetic field; and

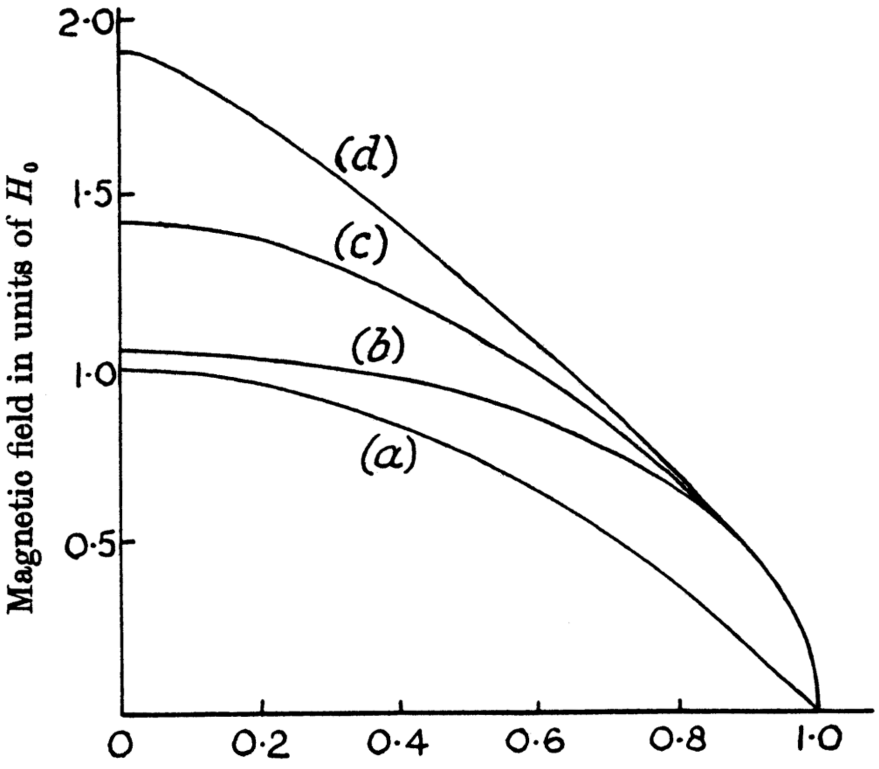
2) The diamagnetic energy of the excluded magnetic field (curve b);

wherein the Gibbs potential is the resultant curve c, and the minimum value ω' is the Gibbs potential at H_c.

Magnetic hysteresis arises when there is an increase in free energy along curve c between 0 and ω', creating a potential barrier to the phase transition.

Superheating in a Mesoscopic Type I Superconductor

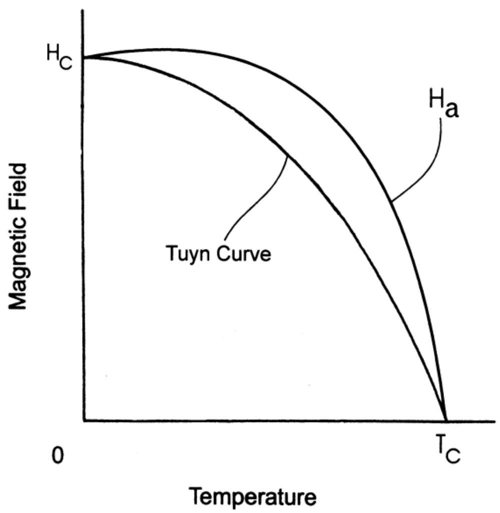
Fig. 6



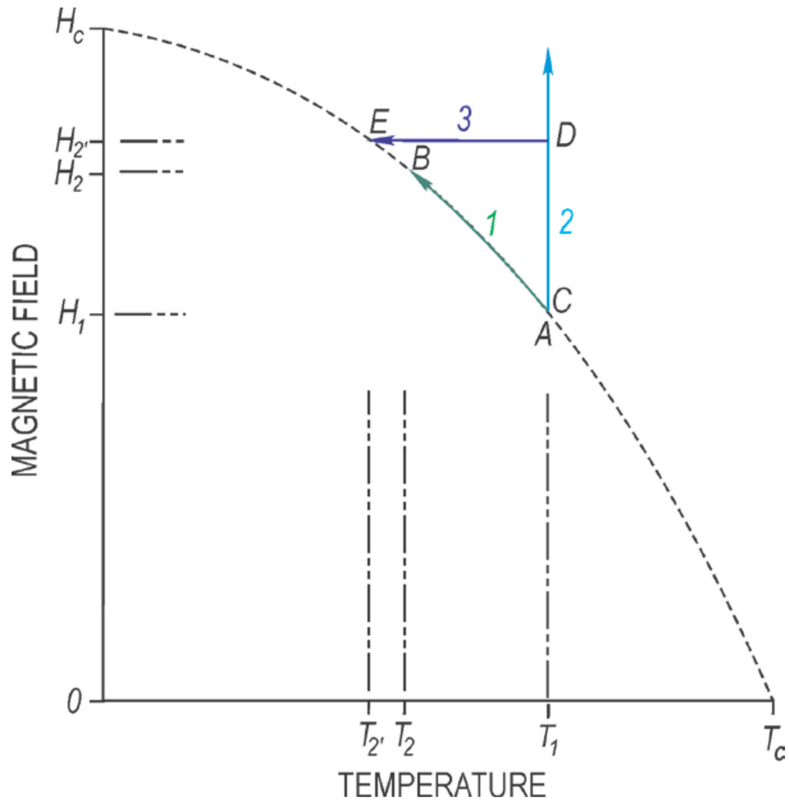
Critical fields versus reduced temperature :
(a) H_c for bulk material,
(b) H_{sc} for colloidal sphere of radius $3\lambda_0$,
(c) H_e for colloidal sphere of radius $3\lambda_0$,
(d) H_{sh} for colloidal sphere of radius $3\lambda_0$.

Taken from A. B. Pippard, Phil. Mag., 43, 273 (1952)

Pippard predicts that critical field hysteresis is maximum at absolute zero, and disappears at finite temperature, well below T_c. This result is in fatal conflict with Bardeen's Hypothesis.



Experiment Theory



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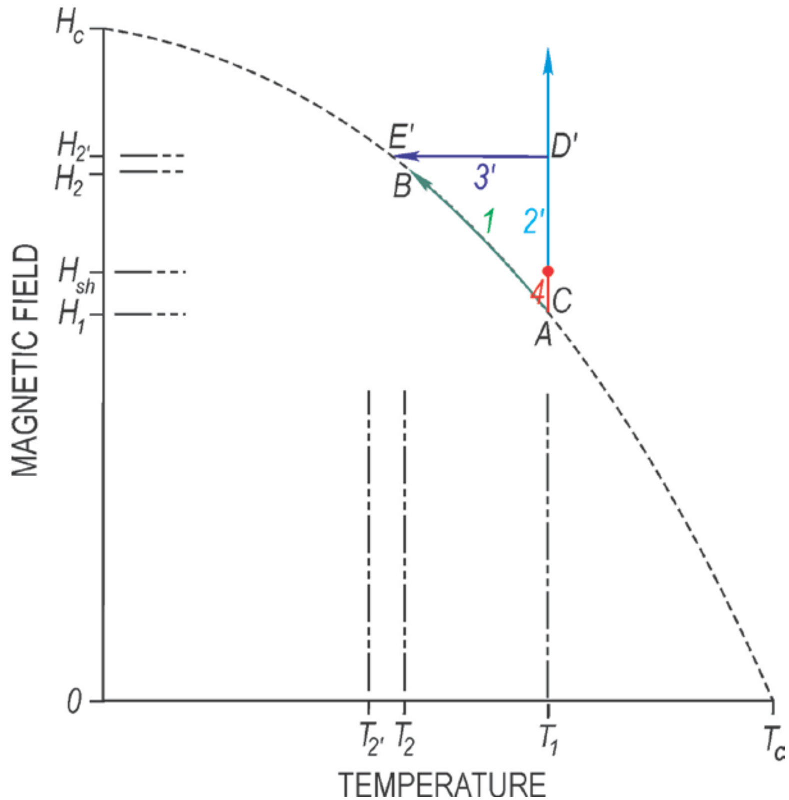
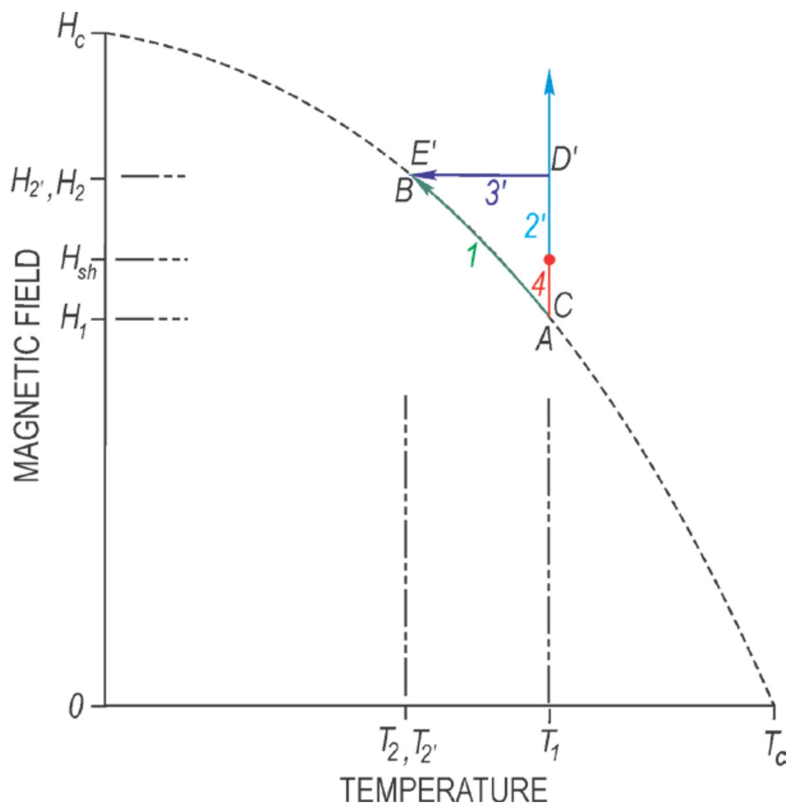
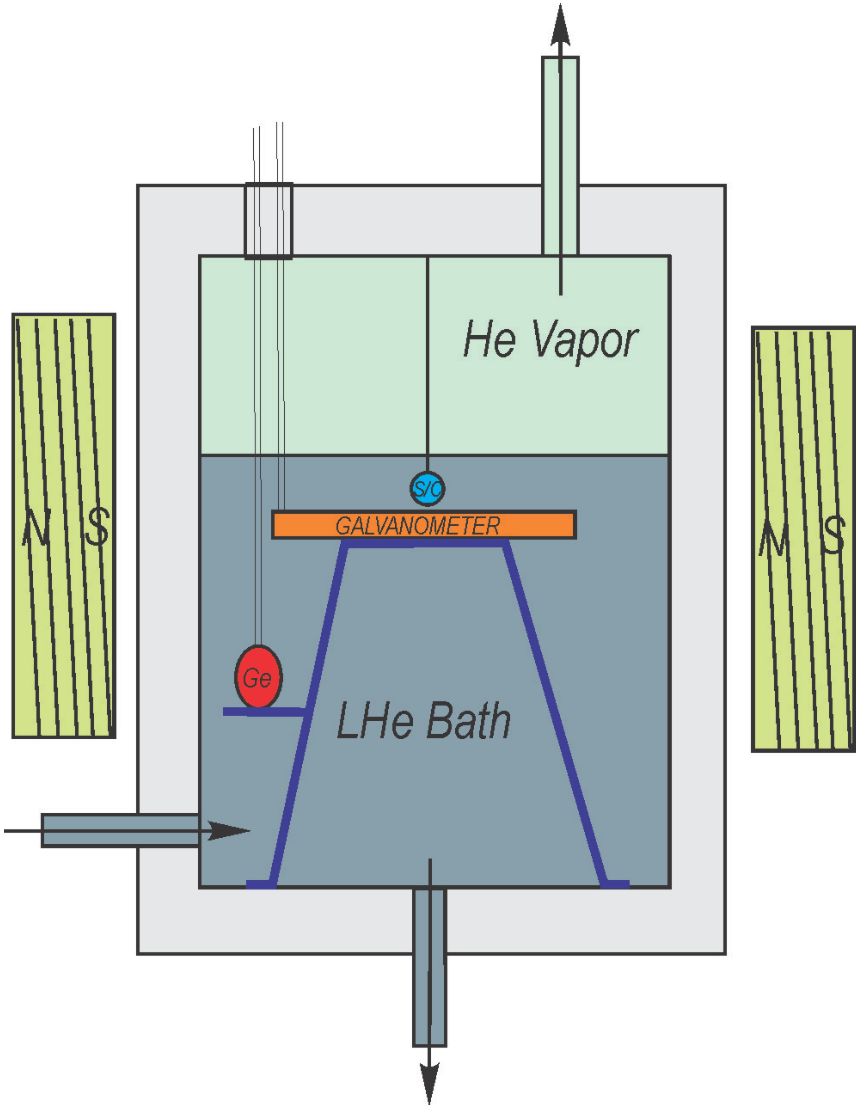


Experiment Theory



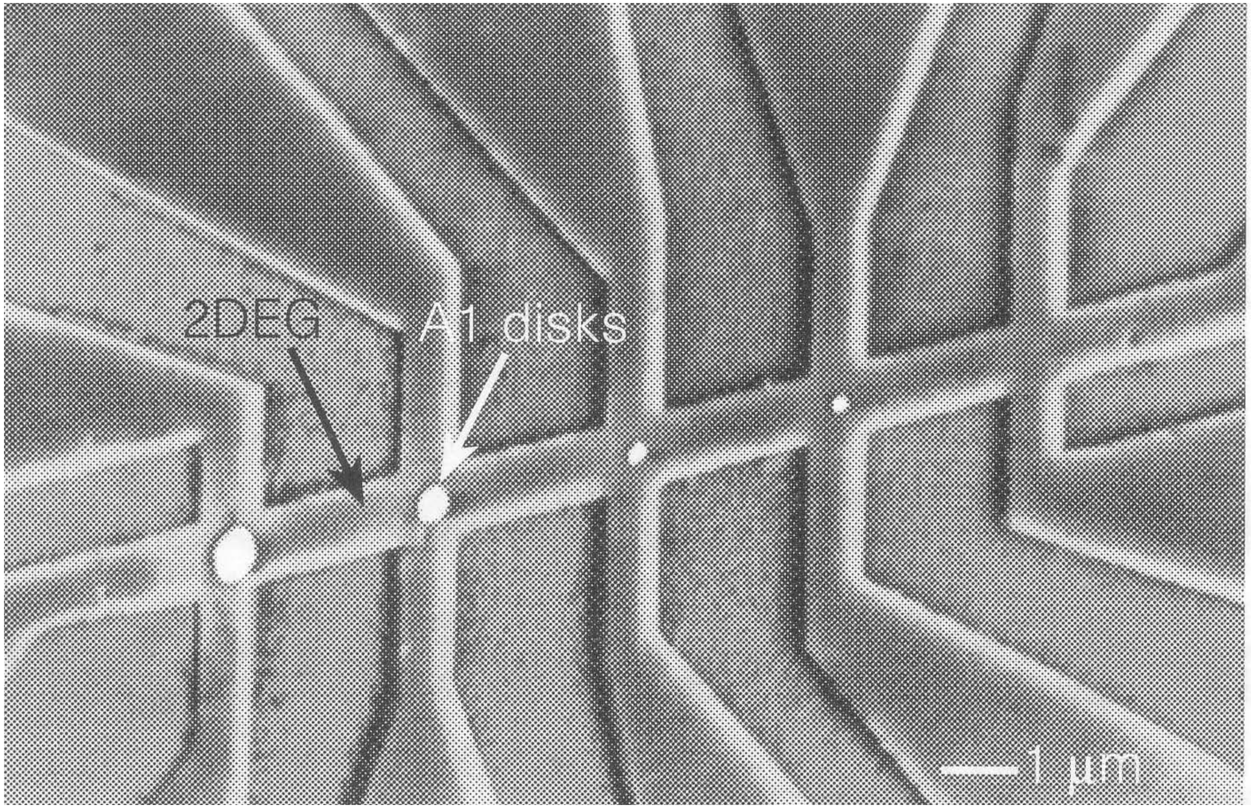
Experimental Set-Up



Ballistic Hall Micromagnetometer

A magnetization measurement device which allows quantitative studies of thermo-dynamic properties of individual submicron superconducting particles fabricated on a GaAs/GaAlAs heterostructure with a high-mobility two-dimensional electron gas (2DEG) embedded 60 nm below the surface. A confined ballistic electron flow passes in close vicinity of a small magnetized object, deviations of the beam due to a stray magnetic field are detected. The signal depends on the filling factor at the junctions.

A.K. Geim, S.V. Dubonos, J.G.S. Lok, I.V. Grigorieva, J.C. Maan, L. Theil Hansen and P.E. Lindelof, Appl. Phys. Lett. 71, 2379 (1997).



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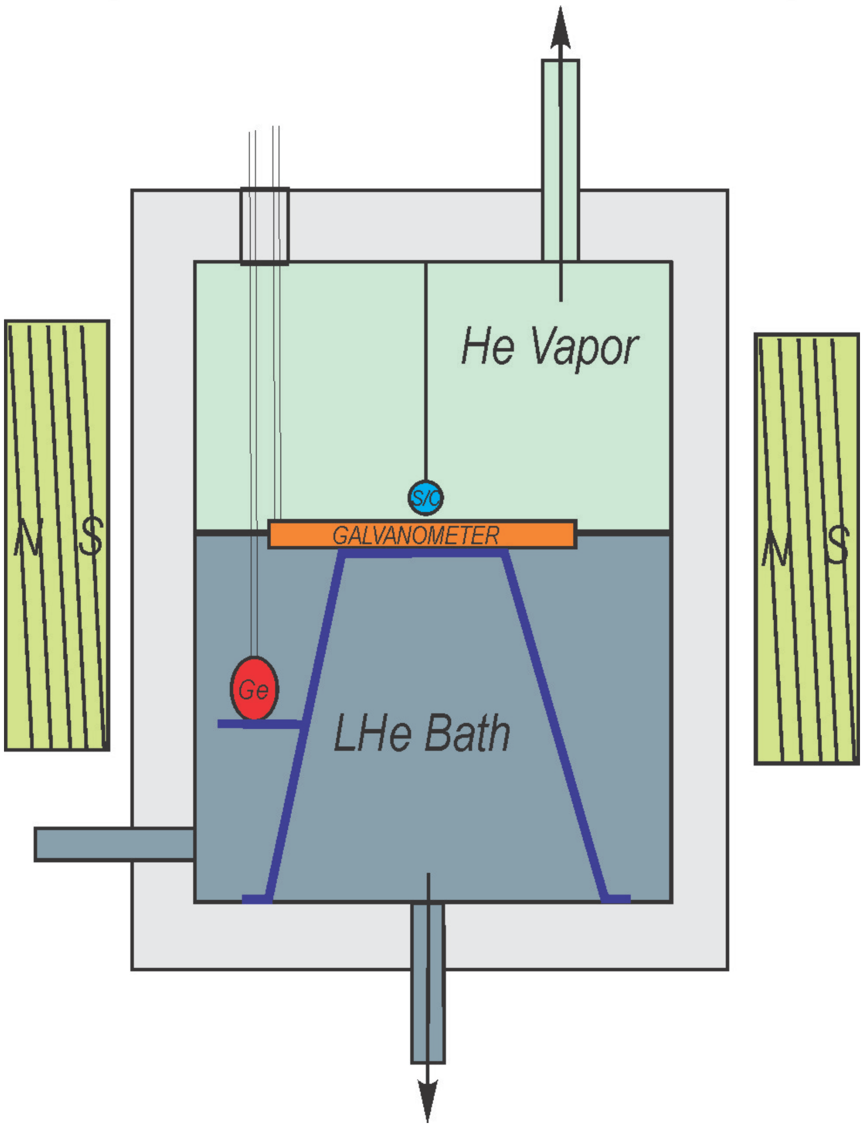

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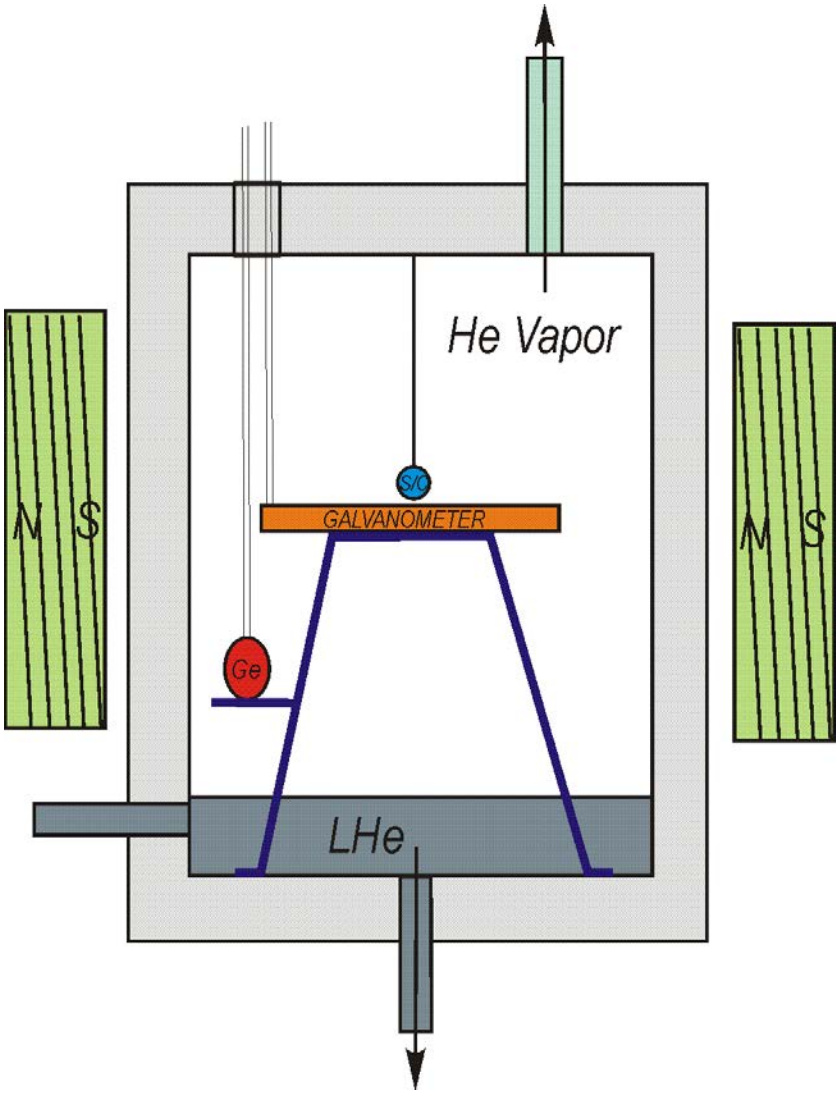

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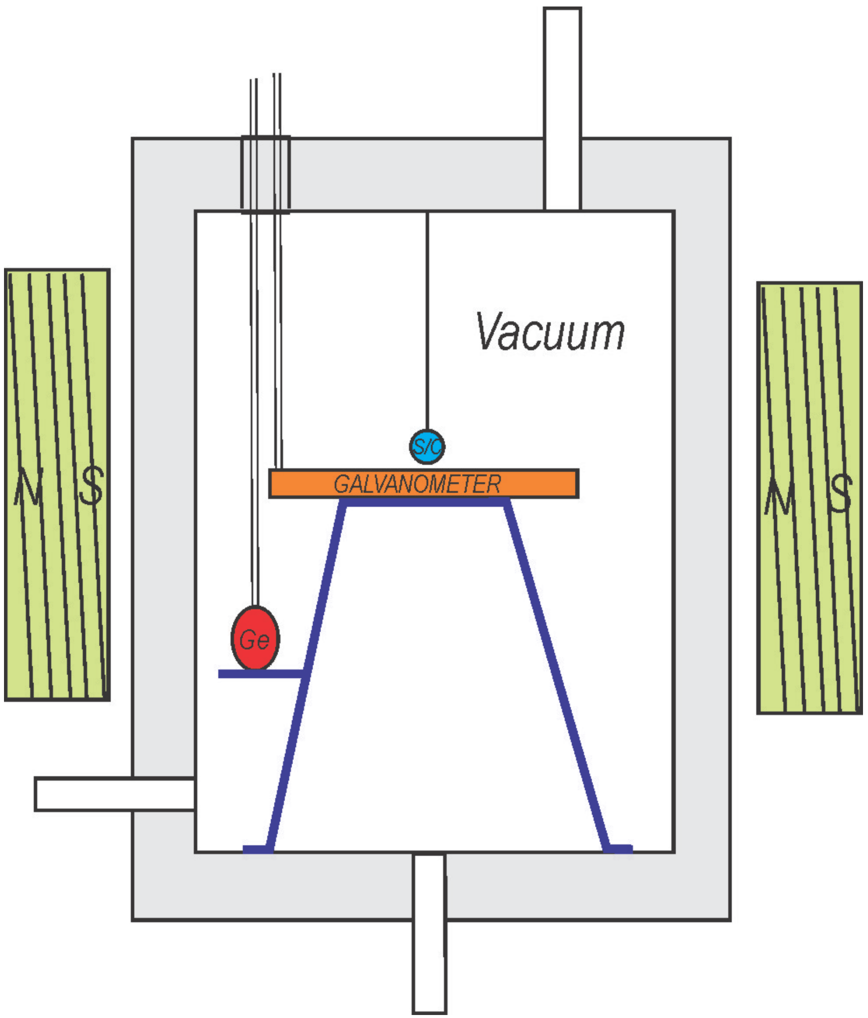
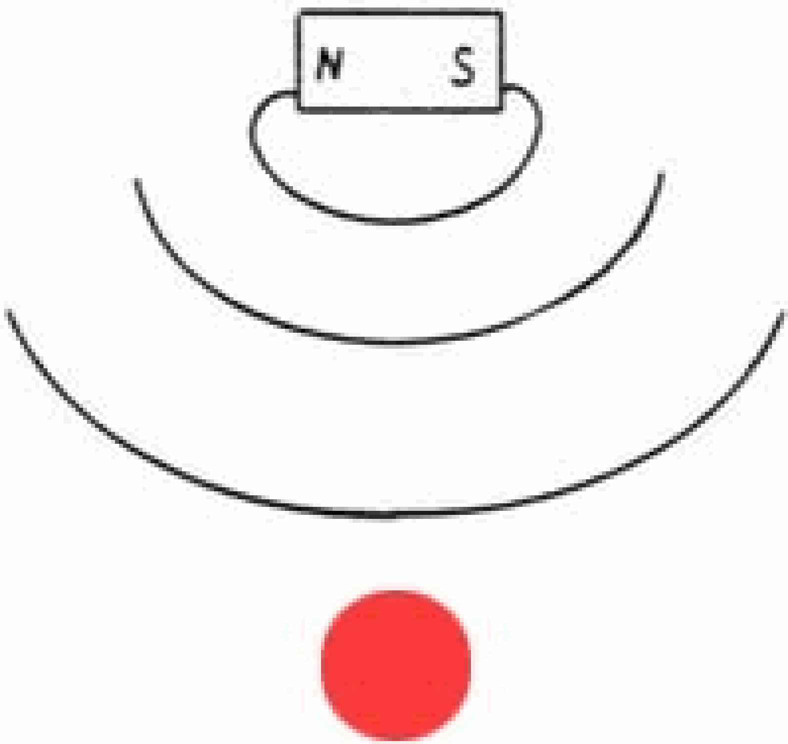
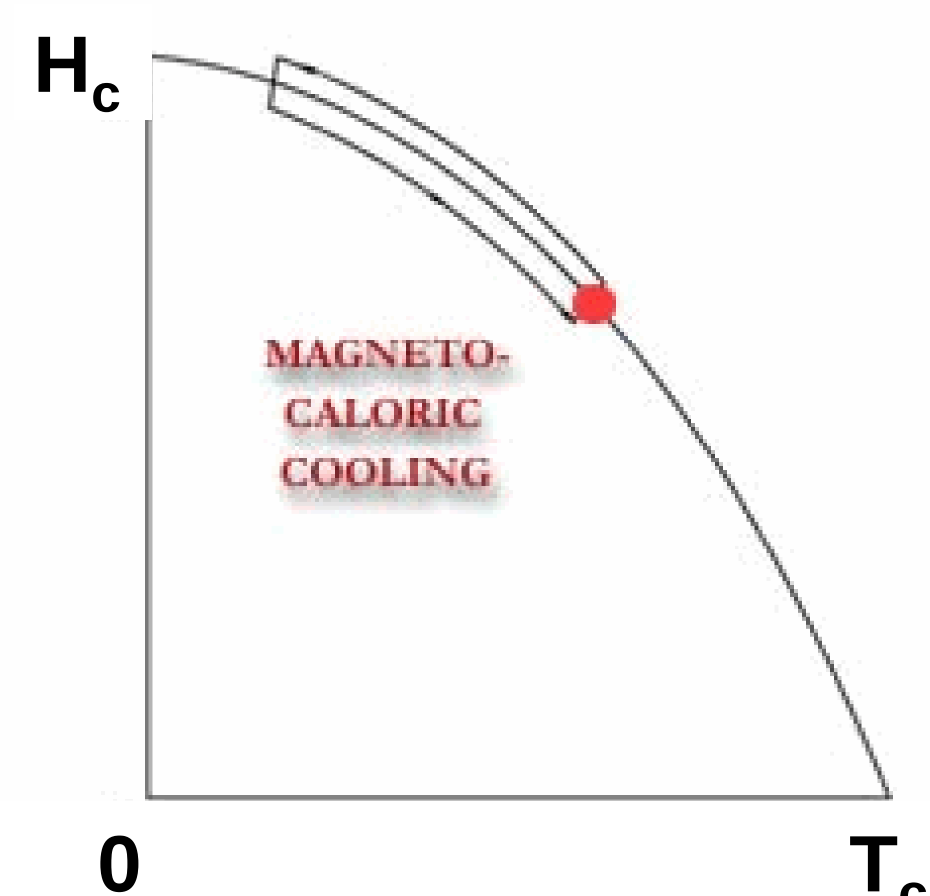


Thank You!

Keefengine.com



Work Cycle



H-T Space Cycle